# The Basic Scheme of the Afocal System and Composition Variants of the Objectives Based on It

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Abstract. The flat refractive surface allows to achieve a perfect spherical aberration correction, so it can be used in different optical systems. For example, the parallel-sided plate can be used for the correction of spherical aberration of the spherical mirror. Similarly it can be applied in the afocal system consisting of the first concave and the second convex mirrors. In such a system the plate thickness is rather big. So the afocal system of two spherical mirrors and the plate was modified: the second mirror was combined with the second surface of a plate. In the afocal system which consist of a concave mirror and an catadioptric planoconcave lens the spherical aberration correction can be achieved by the variation of he lens thickness. The long-focus objective can be built by combination of that scheme and the additional lens system. The possibility of aberration correction in such systems is considered.

## 1. Introduction

The catadioptric system are widely used for different purposes, for example as photographic objectives, collimators, objectives for instruments night vision and others. Different variants of such objectives exists, but new tasks are appearing, so it's necessary to develop new systems. Here one of the possible variant of catadioptric objective is offered.

## 2. Afocal catadioptric system

The parallel-sided plate placed in converging beam was proposed for the compensation of spherical aberration [1, 2]. Such a system may be used for the mirror testing.

Similarly the plate can be used for the spherical aberration correction in the two-mirror afocal system [3]. But in these systems the plate thickness is great, that's why another scheme was proposed [3]: the system where the second mirror is on the second plate surface (figure 1).

The third order spherical aberration of the system shown in figure 1 is corrected if the plate thickness  $d_2$  is equal:

$$d_2 = \frac{n}{\alpha} \left( \frac{n^2 - \eta}{4(n^2 - 1)} - \eta \right),$$

and the distance between the firs mirror and the plate  $d_1$  (see figure 1):

$$d_1 = \frac{3n^2 + \eta - 4}{4\alpha(n^2 - 1)}$$

Here  $\eta$  – an obscuration in the system, which is equal to the diameters ratio of the mirrors:  $\eta = D_2/D_1$ ; n – refraction index of the plate glass;  $\alpha = 1/f_1'$ ,  $f_1'$  - the focal distance of the first mirror.



Figure 1. The afocal catadioptric system.



**Figure 2.** The graphic dependence for the plate thickness  $d_2$  of the refraction index n for the different obscuration ratio  $\eta = 0.1$  and  $\eta = 0.3$ .

The graphics of plate thickness dependence of the refraction index are displayed in figure 2. The thickness is scaled to the focal distance of the first mirror.

It's obviously the plate thickness can be changed in very wide range by the selection of the refraction index and the obscuration ratio.

If the spherical aberration is corrected coma is determined by the coefficient  $K_0$  [4,5], in this case is equal:

$$K_0 = -\frac{\alpha^2 (n^2 - 1)}{4n^2}$$

and astigmatism and field curvature are determined by the coefficients C0 and D0 respectively

$$C_0 = \frac{\alpha}{4n^2\eta} [\eta(2-3n^2)+1], \ D_0 = \frac{\alpha(\eta n^2 - 1)}{\eta n^2}$$

#### 3. Catadioptric objectives

Afocal system with the corrected spherical aberration may be combined with additional optical system, so that the whole system are corrected for spherical aberration. In such a combination different aberration correction may be achieved. Not only spherical aberration, but coma, astigmatism and field curvature in some cases. The possibility of aberration correction depends on the configuration of the additional optical system. Several variants of composition are given below.

#### 3.1. Afocal system with the thin component

Let the objective is composed of the afocal catadioptric system and the additional objective -a system of two glued lenses or two unglued lenses, which is placed near the firs mirror (figure 3).

In the whole objective the spherical aberration is corrected if the additional system has no spherical aberration. But in the whole objective aplanatic correction can be achieved by the appropriae choice of the additional component parameters.

In the aplanatic objective astigmatism is defined by the coefficient

$$C_0 = \frac{\alpha[-\eta^2 n^2 + \eta - 2(n^2 - 1)]}{4\eta^2 n^2} + \frac{1}{\eta^2}$$

where  $\alpha = \frac{f'}{f_1'}$ , f' – focal distance of the whole objective, f'<sub>1</sub> – focal distance of the first mirror



Figure 3. The layout of the catadioptric objective.

Last expression shows that astigmatism is corrected, if

$$\alpha = -\frac{4\eta n^2}{-\eta^2 n^2 + \eta - 2(n^2 - 1)}$$

For the refraction index in the range n = 1.2 - 2 and obscuration in the range  $0 \div 0.3$  the angle  $\alpha > 0$ , so he astigmatism couldn't be corrected.

The catadioptric aplanatic objective is shown in figure 4. It was designed preliminary by the obtained formulas, and then was optimized; the objective is corrected for the spherical aberration, coma, chromatic aberration of position; field curvature is not big, but astigmatism is large. With the vignetting of 50% the spot diameter is not greater than 0.40 mm through the whole field (linear field in image space of the objective 2y' = 20 mm). In table 1 values of tangential and sagittal shifts are given.

Table 1. Sagittal and tangential shifts for the aplanatic catadioptric objective in figure 4.

Image size	Tangential shift	Sagittal shift	Astigmatism
(mm)	$z'_m$ (mm)	$z'_{s}$ (mm)	$z'_m - z'_s \pmod{m}$
0	0	0	0
7.12	-2.06	-0.787	-1.273
10.1	- 4.23	- 1.6	-2.63





**Figure 4.** Catadioptric aplanatic objective f' = 500 mm, 1:5, 2y' = 20 mm.

**Figure 5.** Catadioptric anastigmatic objective f' = 500 mm, 1:5, 2y' = 20 mm.

If the spherical aberration is not corrected in components (i.e. in the afocal system and the additional objective), the astigmatism correction can be fulfilled in the objective. Let the plate thickness is defined by the expression:

$$d_{2} = \frac{1}{\alpha \nu} \left( \frac{1 - \nu^{2} \eta}{4(1 - \nu^{2})} - \eta \right) + \delta$$

where  $\delta$  defines the imperfect spherical aberration correction in the afocal system; v = 1/n. Let the auxiliary value *x* is:

 $x = 4\delta\alpha\nu(1-\nu^2)$ 

Then the aplanatic correction is achieved by the appropriate choice of the additional component parameters, and the astigmatism is corrected if the angle  $\alpha$  is determined by the following expression:

$$\alpha = -\frac{4\eta^3}{[\nu^2\eta - \eta^2 - 2(1 - \nu^2)]\eta^2 + x}$$

The angle  $\alpha$  is negative ( $\alpha < 0$ ) if x > 0, i.e. the plate thickness need to be increased, then the astigmatism of the aplanatic objective can be corrected.

Anastigmatic objective of this type is shown in figure 5. With the vignetting of 50% the spot diameter is about 0.05 mm for the edge of field. In table 2 values of tangential and sagittal shifts are given.

Image size (mm)	Tangential shift $z'_m$ (mm)	Sagittal shift $z'_{s}$ (mm)	Astigmatism $z'_m - z'_s$ (mm)
0	0	0	0
7.082	-0.269	-0.251	-0.018
10	-0.494	-0.494	0

Table 2. Sagittal and tangential shifts for the anastigmatic catadioptric objective in figure 5.

That objective is anastigmatic, but the field curvature is not equal to zero, so it can be corrected by the Smith lens. That system variant is shown at figure 6. With the vignetting of 50% the spot size is about 0.02 mm for the edge of field. In table 3 values of tangential and sagittal shifts are given.



Figure 6. Catadioptric anastigmatic objective with the Smith lens f' = 500 mm, 1:5, 2y' = 20 mm.

**Table 3.** Sagittal and tangential shifts for the anastigmatic catadioptric objective with the Smith lens in figure 6.

Image size	Tangential shift	Sagittal shift	Astigmatism
(mm)	$z'_m$ (mm)	$z'_{s}$ (mm)	$z'_m - z'_s \pmod{m}$
0	0	0	0
7.11	-0.0273	-0.0078	-0.0195
10.1	- 0.0409	-0.011	0.0299

#### 3.2. Objectives with the afocal compensator

Another way to obtain an anastigmatic objective is to use an afocal compensator in the aplanatic objective. The objective obtained by the adding of the afocal compensator to the system in figure 4 is shown. With the vignetting of 50% the spot diameter is about 0.12 mm for the edge of field. In figure

8 the objective with the Smith lens and the afocal compensator is shown. In tables 4 and 5 values of tangential and sagittal shifts for the catadioptric objectives are given



Figure 7. Catadioptric aplanatic objective with the afocal compensator f' = 500 mm, .1:5, 2y' = 20 mm.



**Figure 8.** Catadioptric aplanatic objective with the afocal compensator and Smith lens f' = 500 mm, 1.5, 2y' = 20 mm.

**Table 4.** Sagittal and tangential shifts for the catadioptric objective with the afocal compensator in figure 7.

Image size	Tangential shift	Sagittal shift	Astigmatism
(mm)	$z'_m$ (mm)	$z'_{s}$ (mm)	$z'_m - z'_s \pmod{m}$
0	0	0	0
7.27	- 1.03	-0.474	-0.561
10.7	-0.722	-0.6736	-0.0491

**Table 5.** Sagittal and tangential shifts for the catadioptric objective with the afocal compensator and Smith lens in figure 8.

Image size (mm)	Tangential shift $z'_m$ (mm)	Sagittal shift $z'_s$ (mm)	Astigmatism $z'_m - z'_s$ (mm)
0	0	0	0
7.34	-0.8	- 0.196	-0.605
11.1	-0.0445	0.008	0.0365

## 3.3. Afocal system with teleobjective

The possible way to construct the objective where astigmatism and field curvature are corrected is to compose an afocal system and teleobjective (positive and negative lens components located at some distance one from another). Possibilities of aberration correction of the obtained system are depended on the balance of coma and spherical aberration of components formed the teleobjective. The schematic layout of objective of that type are shown in figure 9.

The coefficient  $D_0$  in this system, which describe the field curvature:

$$D_0 = \frac{\alpha(\eta n^2 - 1)}{\eta n^2} + \frac{1 - \varphi_1^2 \eta d}{n\eta(1 - \varphi_1 d)}$$

If the optical power of the first component  $\varphi_1$  is given then the distance *d* between components which allow field curvature correction:

$$d = \frac{n + \alpha(n^2 \eta - 1)}{\alpha(n^2 \eta - 1) + \eta n \varphi_1} \cdot \frac{1}{\varphi_1}$$

With  $\alpha = -3.33$ ,  $\eta = 0.3$  and  $\varphi_1 = 3.5/f'$  the distance d = 0.276f' and  $\varphi_2 = -5.114/f'$ , (f' – focal length of the objective).



mm, .1:5, 2y' = 20 mm.

Let us consider the astigmatism correction in the objective. Let the spherical aberration is corrected in each component. If the second component is aplanatic, that is coma is corrected too, then the coma of the whole objective is removed by the first component, and astigmatism is eliminated if the distance is

$$d = \frac{\alpha [n^2 \eta^2 - \eta + 2(n^2 - 1)] - 4n^2 \eta}{\alpha [n^2 \eta^2 - \eta + 2(n^2 - 1)] - 4n^2 \eta^2 \varphi_1} \cdot \frac{1}{\varphi_1}$$

If the angle  $\alpha = -3.333$ , the optical power  $\varphi_1 = 3.5/f^2$ , the obscuration  $\eta = 0.3$  the distance  $d = 0.282f^2$ , the optical power of the second component  $\varphi_2 = -14.54/f^2$ . So it's difficult to eliminate astigmatism in practice.

If the first component is aplanatic, then the coma of the whole objective is removed by the second component, and astigmatism is eliminated if the distance is

$$d = \frac{\alpha [n^2 \eta^2 - \eta + 2(n^2 - 1)] - 4n^2 \eta}{2\alpha (n^2 - 1) + \alpha \varphi_1 [n^2 \eta^2 - \eta + 2(n^2 - 1)] - 4n^2 \eta^2 \varphi_1^2}$$

If the angle  $\alpha = -3.333$ , the optical power  $\varphi_1 = 3.5/f^2$ , the obscuration  $\eta = 0.3$  the distance d = 0.231, the optical power of the second component  $\varphi_2 = -0.873/f^2$ .

Astigmatism and field curvature is corrected if the optical power of the first component:

$$\varphi_1 = \frac{1}{2\eta} + \sqrt{\frac{1}{4\eta^2} + \frac{2(n^2 - 1)[n + \alpha(\eta n^2 - 1)]}{n\eta[\eta(1 + 4n)(\eta n^2 - 1) + 2(n^2 - 1)]}}$$

If the angle  $\alpha = -3.333$  and the obscuration  $\eta = 0.3$  the optical power  $\varphi_1 = 4.616/f'$ , воздушный промежуток d = 0.172f', сила второго компонента  $\varphi_2 = -6.23/f'$ ; if  $\alpha = -1.5$  and  $\eta = 0.3$  the optical powers  $\varphi_1 = 5.155/f'$ ,  $\varphi_2 = -4.43/f'$ , the distance d = 0.178.

The objective obtained by the composition of the afocal system and teleobjective is shown in figure 10. With the vignetting of 50% for the edge of field the spot diameter is about 0.02 mm. In table 6 values of tangential and sagittal shifts are given.

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	Image size	Tangential shift	Sagittal shift	Astigmatism	
	(mm)	$z'_m$ (mm)	$z'_{s}$ (mm)	$z'_m - z'_s \pmod{mm}$	
	0	0	0	0	
	7.08	-0.0078	-0.0028	-0.005	
_	10	-0.000553	-0.0015	0.00095	

Table 6. Sagittal and tangential shifts for the catadioptric objective in figure 10.

#### 4. Conclusion

Using the catadioptric component with the internal reflection as the second mirror in the afocal system allows to obtain a system with the perfect correction of spherical aberration, and also to design objectives with the aplanatic and anastigmatic correction and small field curvature. These objectives can be used as long focus objectives for example in collimators and as photographic objectives.

### References

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